Precoding for Point-to-Multipoint Transmission over MIMO ISI Channels

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- MIMO ISI channel
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- Spectral factorization
- Simulation results
- Summary
**Situation:** Transmission from a central transmitter (e.g., base station) to scattered receivers (e.g., mobile terminals)

\[ \Rightarrow \textit{point-to-multipoint transmission (down link)} \]

- Channels which produce intersymbol interference (ISI)

\[ \Rightarrow \textit{multiple-input/multiple-output (MIMO) ISI channels} \]

**Problem:** Intersymbol and multi-user interference

\[ \Rightarrow \textit{need for separation/equalization of the users' signals} \]
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Problem: Intersymbol and multi-user interference

⇒ need for separation/equalization of the users’ signals

Here: Nonlinear joint preprocessing of the users’ signals at the transmitter side

⇒ precoding

⇒ counterpart to decision-feedback equalization/
successive cancelation in multipoint-to-point schemes
MIMO ISI Channel

System model:

- $N_R$ distributed receivers, each with one antenna
- Central transmitter with $N_T \geq N_R$ transmit antennas; square-root Nyquist transmit filter $T \cdot h_T(t) \circ T \cdot H_T(f)$
- Continuous-time impulse responses $h_{C,\mu,\nu}(t) \circ H_{C,\mu,\nu}(f)$ spatially and temporally white additive Gaussian noise at receive antennas
Transmission over ISI channels:

- Optimum receiver for PAM transmission over ISI channels [Ericson 1971];
  generalization to MIMO ISI channels [van Etten 1975/76]:
  - Matched filter  - $T$-spaced sampling  - discrete-time processing

- Optimum transmission strategy: (dual to procedure above)
  - Joint discrete-time processing
  - Matrix of matched filters for $H_C(f) = [H_{C,m,\nu}(f)]$ at transmitter
  - Matched filters for $H_T(f)$ at receiver
  - $T$-spaced sampling
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- Discrete-time channel model:

$$H_o(e^{j2\pi fT}) = \sum_{l=-\infty}^{+\infty} H_T^*(f + l/T) H_C(f + l/T) H_C^*(f + l/T) H_T(f + l/T)$$

$$\Phi_{nn}[\kappa] = \mathbb{E}\{n[k + \kappa] n^H[k]\} \circ \Phi_{nn}(e^{j2\pi fT}) = \sigma_n^2 I \quad \text{with} \quad \sigma_n^2 = \frac{N_0}{T}$$
Abbreviation: $r_{\nu,\mu}(t) = h_{C,\mu,\nu}^*(-t)$
Precoding for MIMO ISI Channels

Spatial/temporal Tomlinson-Harashima precoding:

\[ F(z) x[k] + H_0(z) y[k] + n[k] \]

\[ \hat{a}[k] = g I \]

with

**Feedforward filter** \( F(z) \): Shapes the end-to-end signal transfer function

\[ H(z) = \sum_k H_k z^{-k} = H_0(z) F(z) \]

**Feedback filter** \( B(z) - I \): Pre-subtraction of known interference

**Permutation matrix** \( P \): Processing order

**Scaling factor** \( g \): Compensation of transmitter side scaling
Requirements:

- **Causality:**
  
  Temporal causality: End-to-end response has to be causal: \( H(z) = \sum_{k \geq 0} H_k z^{-k} \)
  
  Spatial causality: For successive detection of symbols at one time instant \( H_0 \) has to be lower triangular with unit main diagonal
  
  \( \Rightarrow \) Processing in a zig-zag fashion over time and space
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- **Normalization:**
  
  Fixed total transmit power for each channel realization; normalization of \(F(z)\)
  
  \(\Rightarrow\) Each user experiences an AWGN channel with
  \[\text{SNR} = \frac{\sigma_a^2}{(g^2\sigma_n^2)}\]
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- **Minimum Phase Property and Processing Order:**
  Decisions are based on \( H_0 \), all other terms are (pre-)canceled: “\( H_0 \rightarrow \max \)”
  \( \Rightarrow H(z) \) should be minimum phase, i.e., \( \det(H(z)) \neq 0, |z| \geq 1 \)
  Optimum processing order: permutation matrix \( P \)
Solution by performing a (spectral) factorization:

\[ P^T H_o(z) P = S(z) \cdot \Sigma \cdot S^H(z^{-*}) \]

with

- Real diagonal matrix \( \Sigma = \text{diag}(\varsigma_1, \ldots, \varsigma_{N_T}) \)
- Causal and minimum-phase matrix polynomial \( S(z) = \sum_{k \geq 0} S_k z^{-k} \)
- \( S_0 \) lower triangular with unit main diagonal
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Feedforward matrix filter: \( F(z) = P S^{-H}(z^{-*}) \Sigma^{-1} g^{-1} \)

Normalization: \( g \) such that power constraint is met

End-to-end transfer function: \( H(z) = H_o(z) F(z) = P S(z) \)

Feedback matrix filter: \( B(z) = H(z) \)
Task: Given $H_o(z)$ find $S(z)$, $\Sigma$, and $P$ such that

$$P^T H_o(z) P = S(z) \cdot \Sigma \cdot S^H(z^{-*})$$

Solution: (Scalar) polynomial factorization via (repeated) Cholesky decomposition of a coefficient Toeplitz matrix by F.L. Bauer 1955

- Extension to matrix polynomials by Youla/Kazanjian in 1978
  - No iterative algorithm given
  - Processing order ignored
Spectral Factorization

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Now: Easy-to-use iterative algorithm

- Two-step algorithm
  - First step: $P = I$
  - Second step: Optimal detection order
First step: \textit{Factorization}

\[ H_o(z) = \sum_k H_{o,k} z^{-k} = T(z) T^H(z^{-*}) \]

\textbf{Iterative algorithm:} (lower triangular matrix $\sqrt{X} = \text{Cholesky factor of } X$)

\[
L_{0,0} = \sqrt{H_{o,0}} \\
L_{m,i} = \left( H_{o,m-i} - \sum_{k=0}^{i-1} L_{m,k} L^H_{i,k} \right) L^{-H}_{i,i}, \quad i = 0, \ldots, m-1 \\
L_{m,m} = \sqrt{H_{o,0} - \sum_{k=0}^{m-1} L_{m,k} L^H_{m,k}}
\]

\textbf{Convergence:}

\[ T_k = \lim_{m \to \infty} L_{m,m-k}, \quad k = 0, 1, \ldots, m \]
Second step: Optimal processing order

Idea:
- Spectral factorization is unique up to a unitary matrix
- Only coefficient matrix at time index 0 is of interest

⇒ Optimal processing order can be derived based on $T_0$ from above
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- Spectral factorization is unique up to a unitary matrix
- Only coefficient matrix at time index 0 is of interest

\[ \Rightarrow \text{Optimal processing order can be derived based on } T_0 \text{ from above} \]

Decomposition (can be done via V-BLAST algorithm applied to \( T_0^H \))

\[ P^T \cdot T_0 = R \cdot Q^H \]

with
- \( P \): Permutation matrix
- \( R \): Lower triangular matrix
- \( Q \): Unitary matrix

Using \( D = \text{diag}(r_{1,1}, \ldots, r_{N_R,N_R}) \), \( R = [r_{l,m}] \)

\[ \Sigma = DD^H \quad S(z) = P^T T(z) Q D^{-1} \]
Simulation Results

Parameters:

- \( N_T = 4 \) antennas at the central transmitter, \( N_R = 4 \) decentralized receivers
- Uncoded transmission using 16-QAM
- Fixed short-term transmit power
- Block-fading MIMO ISI channel; averaging over channel realizations
  - I.i.d. complex Gaussian elements of the tap matrices (discrete-time model)
    Average energy of channel is normalized to \( N_T \cdot N_R \)
  - Power-delay profile:
    - Equal-gain test channel
    - Exponentially decaying ("Pedestrian A")
  - Correlations:
    - No correlations (i.i.d. fading coefficients)
    - Transmitter side correlations ("novi 2" [IST-METRA])
- Processing order: optimal or arbitrary
Simulation Results (II)

Constant power-delay profile; \( L = 2, 4, \) and \( 8 \) \( T \)-spaced taps

\[
10 \log_{10}(\bar{E}_b/N_0) \ [\text{dB}]
\]

SER

Precoding, opt. sort
Precoding, no sort
Linear preequalization

\( L = 2 \)
\( L = 4 \)
\( L = 8 \)

Fischer et al.: Precoding for Point-to-Multipoint Transmission over MIMO ISI Channels
Exponentially decaying power-delay profile; $L = 4$

10 log_{10}(\bar{E}_b/N_0) [dB] $\rightarrow$

SER $\rightarrow$

Fischer et al.: Precoding for Point-to-Multipoint Transmission over MIMO ISI Channels
Simulation Results (IV)

Exponentially decaying power-delay profile; $L = 4$; transmitter-side correlations

![Graph showing SER vs. $10 \log_{10}(E_b/N_0)$ for different precoding methods and correlation conditions.](image)

- Precoding, opt. sort
- Precoding, no sort
- Linear preequalization
- Correlations
- No correlations

Fischer et al.: Precoding for Point-to-Multipoint Transmission over MIMO ISI Channels
Digital transmission over MIMO channels with ISI:

Transmitter: discrete-time processing, pulse shaping, channel matched filter
Receiver: pulse shaping matched filter, $T$-spaced sampling

Attractive equalization strategy in point-to-multipoint scenarios:

Spatial/temporal precoding

Requirements and derivation of optimal discrete-time processing

Filter calculation by solving a spectral factorization problem

Efficient factorization algorithm has been given

Simulation results show:

Considerable gains over linear preequalization are possible